A Modified Iterative Method of Orthogonal Grid Analysis

Bidyadhar Basa*, Dr. Lakshmidhar Kar**

*(Department of Civil Engineering, Institute of Technical Education and Research, SOA University, Bhubaneswar, Orissa.)

**(Department of Civil Engineering, Institute of Technical Education and Research, SOA University, Bhubaneswar, Orissa.)

ABSTRACT

For the analysis of orthogonal grids a modified iterative method is developed here to analyse interconnected grids. The method is similar to Kani's method and it helps to perform the analysis in only one set of operation. This is in contrast to moment-torque distribution method or rotation contribution method in which 2n+2 sets of operations are required for the analysis of a grid having n number of internal joints. This method is a modified form of Kani's method, which is generally employed for the analysis of multi storied frame. It makes use of the slope-deflection equation for expressing the flexural rotations in terms of bending moments and also gyration equation relating the angle of twist and torsion.

In the proposed method, displacement contribution corresponding to $6EI\delta/L^2$ is taken into account. Equations are developed to calculate the rotation contribution both in East-West direction and in North-South direction in terms of far end rotation contribution. The equilibrium of each joint in vertical direction yields the governing equation for displacement contribution of each member connected to joint in terms of rotation contribution, fixed end reaction and other displacement contribution. The proposed method is capable of analysing grids with any number of internal joints.

Key Words: displacement contribution, modified iterative method, orthogonal grids, rotation contribution

1. Introduction

Grid structures are structural system, which are extensively used in large span bridges and structures with large span slabs. A grid is subjected to forces normal to it and thus it undergoes rotation along two axes and displacement along the third. As a consequence each member is subjected to bending moment as well as torsion in addition to shearing force. Analysis of grids can be done by semi empirical method [1] or exact method [2, 3, 4]. The exact method of analysis need two sets of non-deflection analysis corresponding to rotation along each direction and 2n sets of deflection analysis for n number of internal joints. As a result these methods are not suitable for hand calculations when the number of internal joints is more.

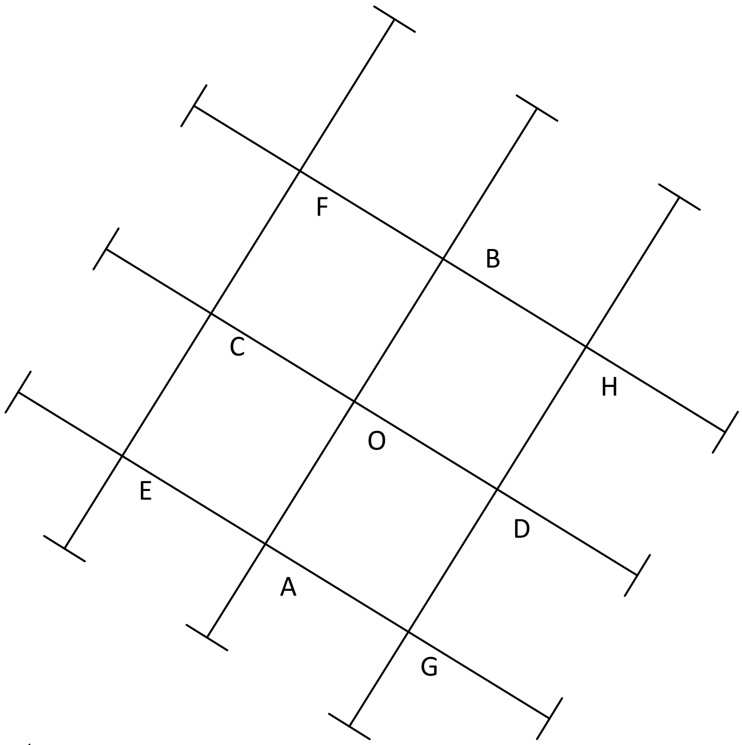
For the analysis of orthogonal grids a modified iterative method is developed similar to Kani's method where sway and non-sway analysis are incorporated into one set of analysis.

2. Development of Method

The equations necessary for solving an orthogonal grid are derived in this section. Equal span grids with one internal joint as well as two and three internal joints lying in one direction are then generalized for the internal joints in both directions. Orthogonal grids of unequal spans are considered next and these equations are further generalized.

2.1 Operational Equation for Rotation Contribution

Following Kani's notation, the end moment (M_{AB}) of any member can be expressed as sum of fixed end moment (M_{FAB}), two times the near end rotation contribution (M'_{AB}), far end rotation contribution (M'_{BA}) and displacement contribution (M''_{AB}).



$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA} + M''_{AB}$
$M_{BA} = M_{FBA} + 2M'_{BA} + M'_{AB} + M''_{AB}$-- (1)

Here, M''_{AB} represents the displacement contribution corresponding to 6EI δ/L^2 where δ is the vertical displacement. Since there are only two basic unknowns in Eq. 1, the rotation contribution and displacement contribution of a member, we need two governing equations for solving it. Of course, for a grid we will have to consider the moment equilibrium in two directions, usually taken as West-East direction and South-North direction. Thus we will need two sets of rotation factors, corresponding to the two directions. These factors depend on the bending stiffness 4EI/L and torsional stiffness GJ/L. Considering grids of equal spans, the rotation factor for bending (along any direction) is –l/(4+2r) and for torsion is –r/(2(4+2r)) where r = GJ/(2EI). Thus for bending along West-East direction (Fig. 1), the

Fig. 1 A General 3 ×3 Grid

Rotation factors at joint O are
$RF_{OA} = RF_{OB} = -1/(4 + 2r)\ and$
$RF_{Oc} = RF_{OD} = -r/(2(4 + 2r))$
Of course, the sum of four rotation factors in any direction is – 0.5. Considering the moment equilibrium at the joint O in West – East direction.

$M_{OA} + M_{OB} + T_{OC} + T_{OD} = 0$ ---(2)
Where $M_{OA} = M_{FOA} + 2M'_{OA} + M'_{AO} + M''_{OA}$
And $T_{OC} = (\theta_{OA} - \theta_{CE})GJ/L$
On simplifying, Eqn. 2 yields
$M'_{OA} = \frac{-1}{4+2r}[M_{FOA} + M'_{AO} + M''_{OA} + M_{FOB} + M'_{BO} + M'_{OB} - r(M'_{CE} + M''_{DG})]$-----(3)

Equation 3 represents the governing equation for determining the rotation contributions at any internal joint of orthogonal grid of equal span.

2.2 Operational Equation for Displacement Contribution of Single Internal Joint Grid.

For bending moment, anti-clockwise moments are taken as positive for West-East bending (West on left hand side and East on right hand side) as well as South – North bending (South on left hand side and North on right hand side). Fig. 2 shows the free body diagram of a four-member grid. Now considering vertical equilibrium at the joint O (Fig. 2), we have

$R_{OA} - R_{OB} + R_{OC} - R_{OD} - R'_{OA} - R'_{OB} - R'_{OC} - R'_{OD} = 0$ ------------------------------(4)

Where R_{OA} etc are reactions due to final moment, i.e. $R_{OA} = (M_{AO} + M_{OA})/L$ etc and R_{OA} is the statically determinate reaction at O of member OA, etc.

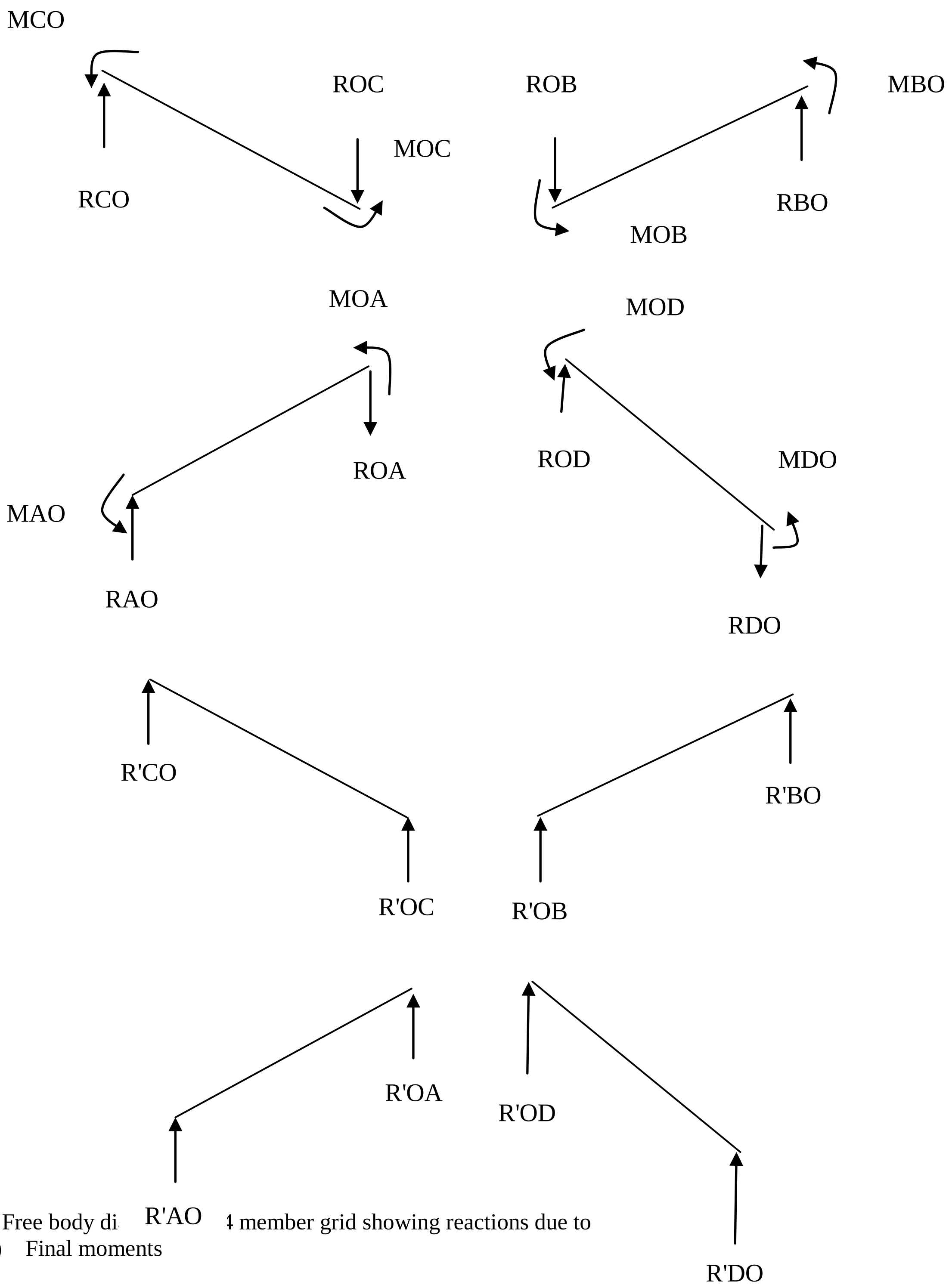


Fig. 2 Free body di 4 member grid showing reactions due to
a) Final moments

b) Given loads

Using the fact that each member has same length and the sign convention adopted, we have (Fig. 3).

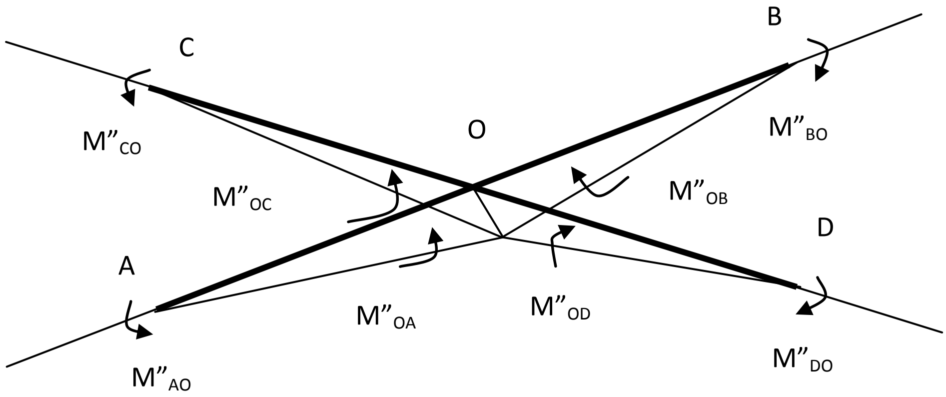


Fig. 3 Fixed end moments due to displacement δ

$M"_{OA} = -M"_{OB} = M"_{OC} = -M"_{OD} = 6EI\delta_1/L^2$---(5)

Substituting the values of end moments in terms of fixed end moments, rotation contribution, displacement contribution in Eq. 4. and using Eq. 5, we get.

$$M"_{OA} = \frac{1}{4}\left[\frac{3}{2}\left(\sum FER_O \frac{L}{3} - \sum (M'_{AO} + M'_{OA})\right)\right]$$

In Eq. 6 is the operational equation for displacement contribution of a single internal joint grid.

$$\sum FER_O = FER_{OA} - FER_{OB} + FER_{OC} - FER_{OD}$$

Eq. 6 is the operational equation for displacement contribution of a single internal joint grid.

2.3 Operational Equation for Displacement Contribution of a Grid Having Two Internal Joints

Let δ_1 and δ_2 be the vertical deflections of joint D and E (Fig. 4). Now we have

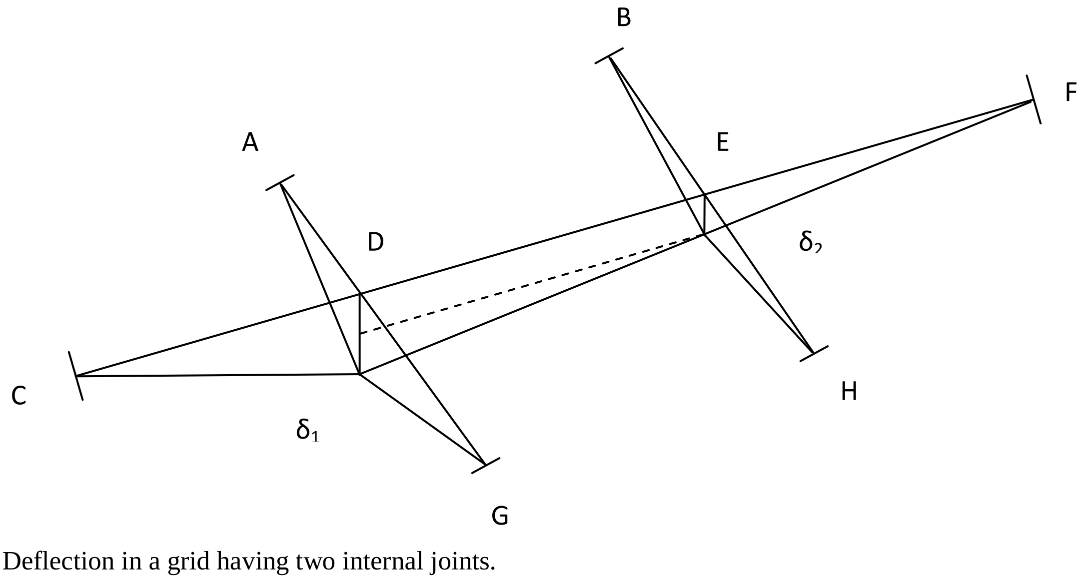


Fig. 4 Deflection in a grid having two internal joints.

$$M''_{AD} = -M''_{DG} = M''_{CD} = 6EI\delta_1/L^2$$
$$M''_{BE} = -M''_{EH} = M''_{EF} - 6EI\delta_2/L^2$$
$$M''_{DE} = -6EI(\delta_1 - \delta_2)/L^2 = -M''_{EF} - M''_{CD}$$

Considering the sign convention developed earlier.

$$\sum_D M'' = M''_{CD} - M''_{DE} - M''_{AD} - M''_{DG} = 4M''_{CD} + M''_{EF}$$

which gives

$$M''_{CD} = \frac{1}{4}\left[\sum_D M'' - M''_{EF}\right]$$

Using Eq. 6 for $\sum_D M''$, we obtain

$$M''_{CD} = \frac{1}{4}\left[\frac{3}{2}\left\{\sum_D FER\frac{L}{3} - \sum (M'_{CD} + M'_{DC})\right\} - M''_{EF}\right] \text{---------------}(7)$$

Similarly,

$$-M''_{EF} = \frac{1}{4}\left[\frac{3}{2}\left\{\sum_E FER\frac{L}{3} - \sum (M'_{DE} + M'_{ED})\right\} + M''_{CD}\right] \text{-------------}(8)$$

$$M''_{DE} = \frac{1}{4}\left[\frac{3}{2}\left\{\sum_D FER\frac{L}{3} - \sum (M'_{CD} + M'_{DC})\right\} - M''_{EF}\right]$$

and

$$M''_{DE} = \frac{1}{4}\left[\frac{3}{2}\left\{\sum_E FER\frac{L}{3} - \sum (M'_{DE} + M'_{ED})\right\} - M''_{CD}\right] \text{---------------}(9)$$

2.4 Operational Equation for Displacement Contribution of a Grid Having 3 Internal Joints in One Direction

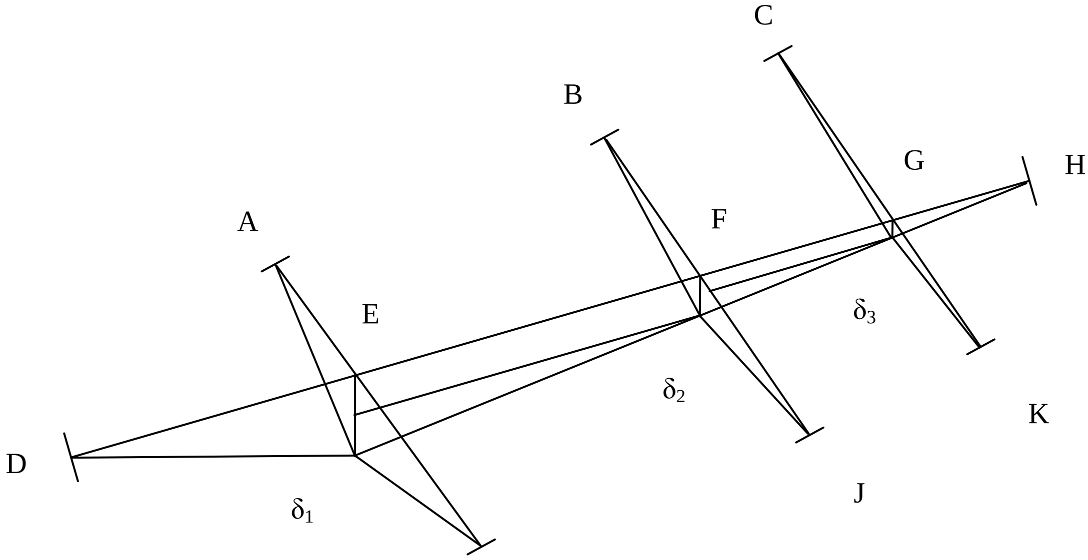


Fig. 5 Deflection in a grid having three internal joints in o rection.

Figure 5 shows a grid having three internal joints E, F and G in one direction. Let δ_1, δ_2 and δ_3 be their vertical deflections. Now,

$$M''_{DE} = -M''_{EI} = M''_{AE} = 6EI\delta_1/L^2$$
$$M''_{BF} = -M''_{FJ} = 6EI\delta_2/L^2$$
$$M''_{CG} = -M''_{GK} = -M''_{GH} = 6EI\delta_3/L^2$$
$$M''_{FG} = -6EI(\delta_2 - \delta_3)/L^2 = -M''_{GH} - M''_{BF}$$

$M"_{EF} = -6EI(\delta_1 - \delta_2)/L^2 = -M"_{DE} + M"_{BF}$

Now

$$\sum_E M" = M"_{DE} - M"_{EF} + M"_{AB} - M"_{EI}$$

which yields

$$4M"_{DE} = \sum_E M" - M"_{GH} - M"_{FG}$$

On simplifying

$$M"_{DE} = \frac{1}{4}\left[\frac{1}{3}\left\{\sum_E FER\frac{L}{3} - \sum (M'_{DE} + M'_{ED})\right\} - M"_{GH} - M"_{FG}\right] ----------(10)$$

and $M"_{EF} = -M"_{DE} + M"_{BF}$

$$= \frac{1}{4}[\frac{3}{2}\left\{\sum_E FER - \sum (M'_{DE} + M'_{ED})\right\} - M"_{GH} - M"_{FG}\Big] - M"_{GH} - M"_{FG} ------(11)$$

Similarly considering joint G

$$M"_{FG} = \frac{1}{4}[\frac{3}{2}\left\{\sum_G FER - \sum (M'_{FG} + M'_{GF})\right\} + M"_{DE} + M"_{EF}\Big] - M"_{EF} - M"_{DE} ----(12)$$

2.5 Generalised Equation for Rotation Contribution and Displacement Contribution

Equation 3 represents the equation for rotation contribution where as Eqn. 6 to 12 represent equations for displacement contribution for grids having internal joints in one direction. These equations can be generalized for grids spanning in both directions. (Fig. 6)

The rotation contribution of any member at any joint is given by

$$M'_{ji} = RF_{ji}\left[\sum_j M_F + \sum_j M'_{ij} + \sum_j M"_{ij} - r(M'_{mi} + M'_{on})\right] ------------(13)$$

Where

j=joint under consideration

M_{ji}=rotation contribution at the joint 'j' of member 'ij'

RF_{ij}=Rotation factor at the end 'j' of member 'ij'

$\sum M_F$=Sum of fixed and moments at the joint 'j' for the two members lying in the 'ij' direction (i.e. $M'_{Fij}+M'_{Fji}$)

$\sum M"_{ij}$=Sum of the displacement contributions of the two members meeting at the joint 'j' in the direction 'ij' (i.e. $M"_{ij}+M"_{kj}$)

M'_{ml} & M'_{on}=Rotation contribution of the members meeting at the joint 'm' and 'o' respectively in the direction 'ij', at the adjacent joint of joint 'j', i.e. m and o; lying in the transverse direction to the direction under consideration.

The displacement contribution of any member can be generalized by

$$M"_{ij} = \frac{(-1)^{p+1}}{4}\left[\frac{3}{2}\left(\sum_{P=1}^{4} FER_j\frac{L}{3} - \sum_{P=1}^{4}(M'_{ij} + M'_{ji})\right) - \sum_{P=1}^{4}(-1)^{p+1}\sum M"_{ih}\right] - \sum M"_{ih} --- (14)$$

Where

p = 1 = Left or West of joint 'j'

2=Right or East of joint 'j'

3=Top or North of joint 'j'

4=Bottom or South of joint 'j'

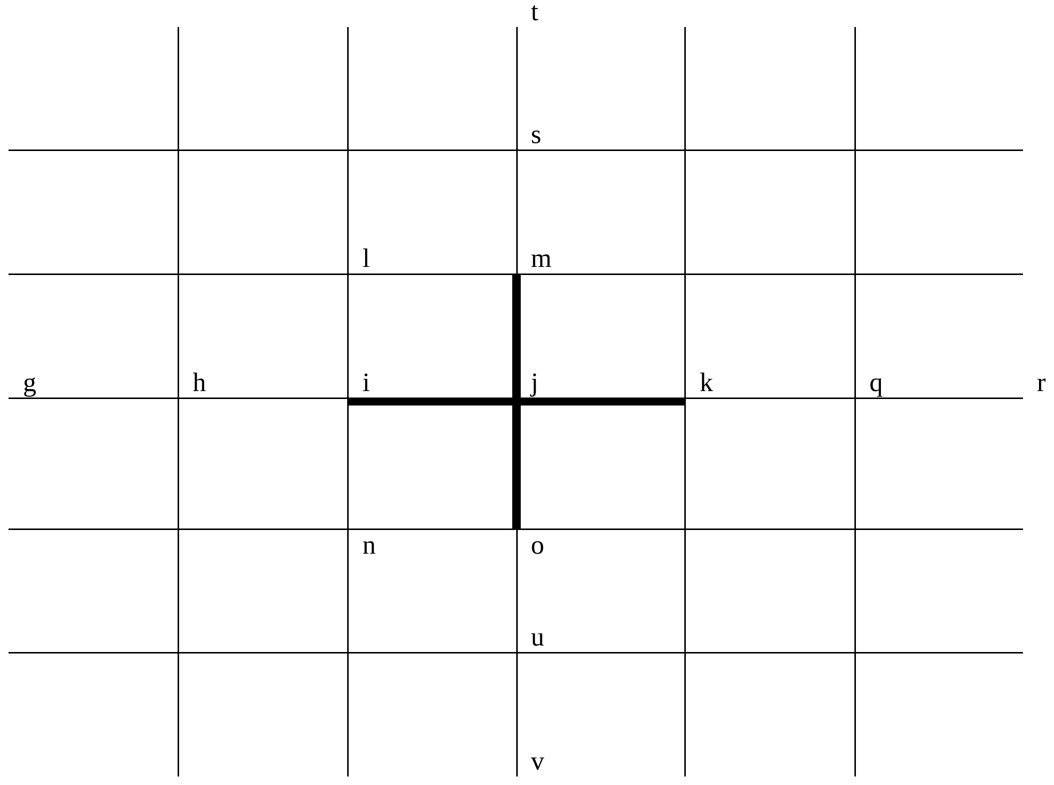


Fig. 6 General Grid

M''_{ij}=Displacement contribution of member 'ij'

$$\sum_{p=i}^{4} FER_j = fixed\ end\ reaction\ at\ joint\ 'j', for\ all\ the\ members\ p = 1\ to\ 4$$

M_{ij}=Rotation contribution at 'i' of member 'ij'
M_{ji}=Rotation contribution at 'j' of member 'ij'
M_{ih}=Displacement contribution of the member present in one side (East/West/North/South) of joint 'j' except for the member meeting at the joint 'j' (i.e. for p=1, $\sum M''_{ih} = M''_{gh} + M''_{hi}$)
It may be noted here that the last term in Eq. 14 vanishes when the joint i or j of member ij is a fixed support (Eq. 6, 7 and 10).

2.6 Operational Equation for Rotation Contribution and Displacement Contribution of a Grid of Unequal Span

For grids of unequal span and unequal rigidities, the equation for rotation contribution is obtained as

$$M''_{ij} = RF_{ji}\left[\sum_j M_F + \sum M'_{ij} + \sum M''_{ij} - \left(\frac{GJ_{mj}}{2EI_{mj}} \times \frac{L_{ml}}{L_{mj}} \times M'_{ml} + \frac{GJ_{Oj}}{2EI_{on}} \times \frac{L_{on}}{L_{oj}} M'_{on}\right)\right] --- (15)$$

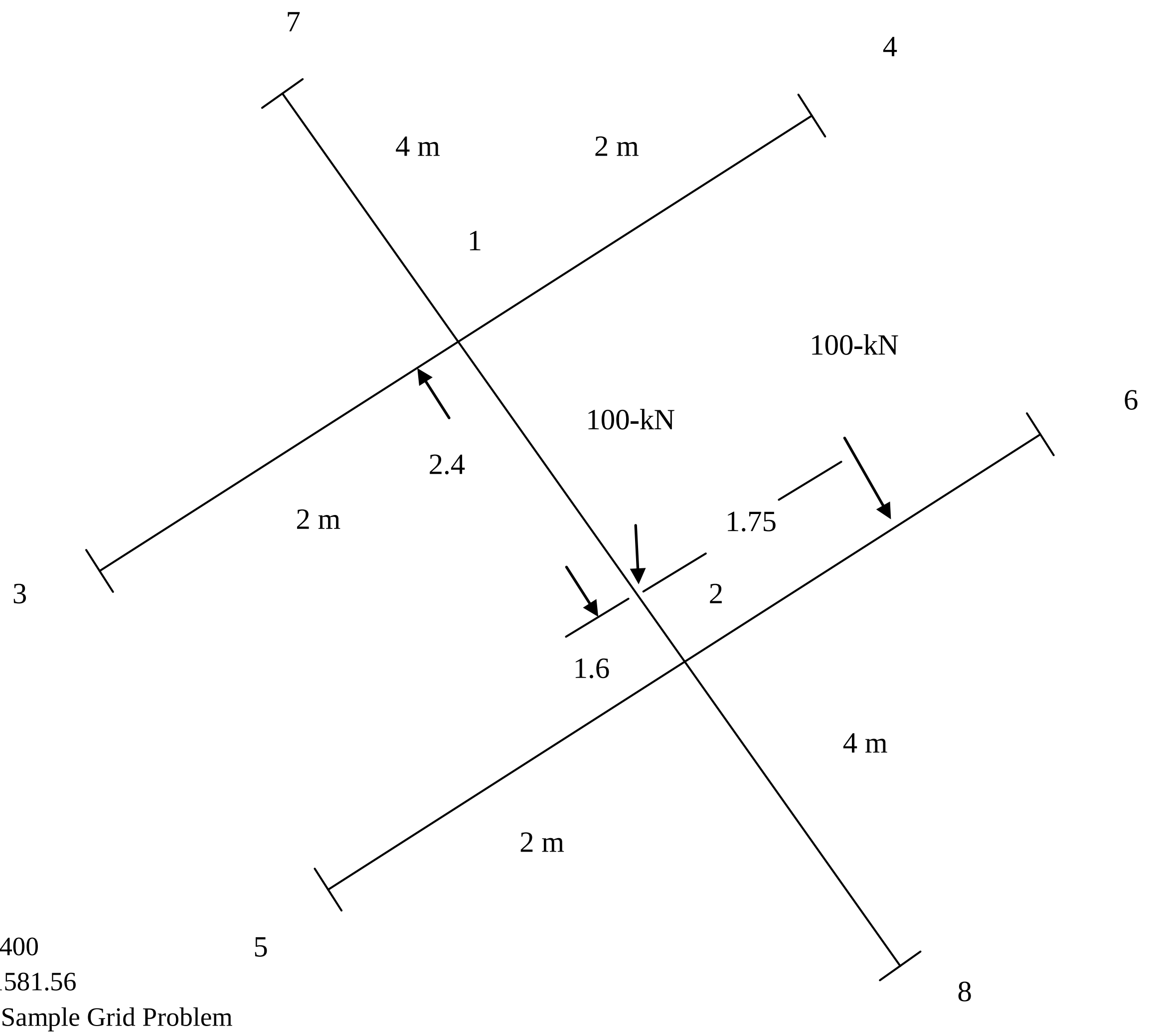
Where GJ_{mj}=GJ value of member mj
$2EI_{ml}$=2EI value of member ml
The equation for displacement contribution also can be obtained as

$$M''_{ji} = \frac{(-1)^{p+1}\{K_{ji} \times C_{ji}\}}{\sum_{p=1}^{4} K_{ji} \times C^2_{ji}} \left[\frac{3}{2}\left(\sum_{p=1}^{4} FER_j \frac{L_r}{3} - \sum_{p=1}^{4} (M'_{ji} + M'_{ij})\ C_{ji}\right) - \sum_{p=1}^{4} (-1)^{p+1} \sum \frac{K_{ji}\, C^2}{K_{ih} C_{ih}} M''_{ih}\right]$$
$$- \sum \frac{K_{ji}\, C_{ji}}{K_{ih} C_{ih}} M''_{ih} ----------(16)$$

Where L_r=Reference length and $C_{ij} = \frac{L_r}{L_{ij}}$
The last term in Eq. 16 vanishes when fixed ends are encountered.

3. Numerical Example

Figure 7 shows a grid having two internal joints and subjected to two point loads, each on span 4 and span 6. First the rotation factors in each direction are determined and kept in enter boxes (-0.2157 and -0.2404). The fixed end moments and reactions are then determined. Sum of fixed end moments in each direction (0, 38.4 at joint 1 and -57.6 and 17.58 at joint 2). Taking L_r=4m, c_{ij} values for member 1, 2, 3 and 4 are 2 whereas those for 5, 6 and 7 are 1 each initially all the rotation contributions and displacement contribution are assumed to be zero. The rotation contribution each joint in both direction are then calculated as per Eq. 16 and then displacement contribution of each member is calculated as per Eq. 17. This process is repeated for five cycles and are presented in Table 1. In Table 1, values paralleled to member are rotation contribution and perpendicular are displacement contribution. The final moments are calculated as per Eq. 1 and are tabulated below.



EI=36400
GJ=11581.56
Fig. 7 Sample Grid Problem

Table 1. Calculation Sheet

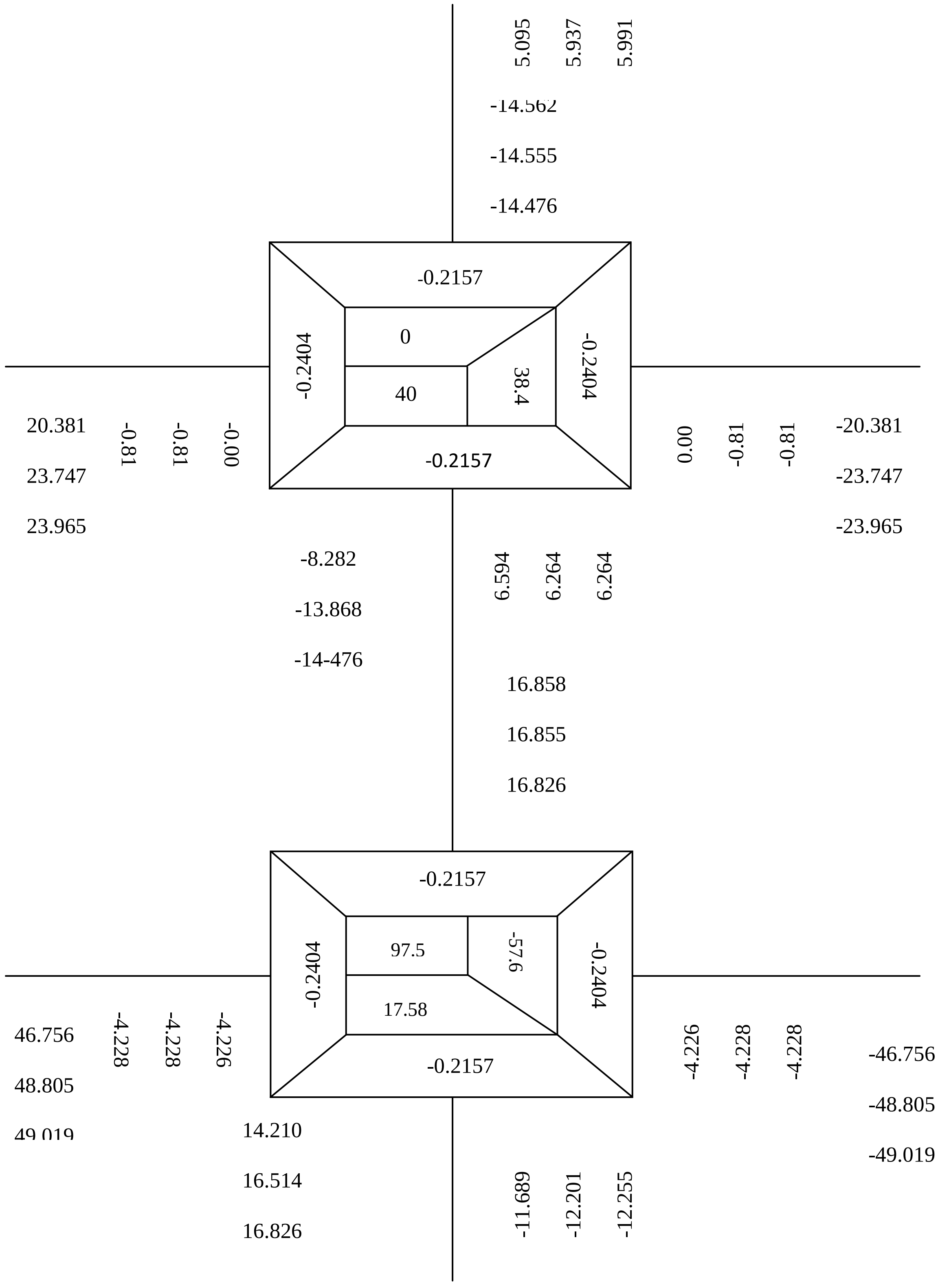


Table 2. Final Moments and Torques

Member	Moment	Torque	Moment	Torque
1	23.908	4.633	23.827	-4.633
2	-24.150	-4.633	-24.069	4.633
3	44.821	-5.633	40.593	5.364
4	-39.927	5.364	82.574	-5.364
5	-8.565	0.006	-23.128	-0.006
6	32.398	0.330	-32.128	-0.330
7	21.453	-0.336	4.596	-0.336

4. Conclusion

A modified iterative method of Orthogonal Grid Analysis has been developed in this work. It is capable of analyzing grids of equal as well as unequal spans and member properties. The major advantage of the proposed method is that it is capable of analyzing a grid in one set of calculation and does not need solution of simultaneous equations. For hand calculation and smaller number of internal joints, this method is most suitable. Its use with the aid of computer is also equally advantageous.

Reference

1. Mathur, G.C. “Analysis of Grid Floors”, The National Building Organisation.
2. Pandit, G.S. and Gupta, S.P., “Moment Distribution in Orthogonal Grid of Interconnected Girders”, *Indian Concrete Journal, Vol. 58 March 1984, No. 3* pp-78-83.
3. Paramsivam, V., Dravid, P.S. and Ramesh, C.K. “Grid Analysis by the Rotation Contribution Method”, *Indian Concrete Journal, Vol. 41, July 1967, No. 7* pp 283-292.
4. Kar, L., Prusty, B.G. and Biswal, D.K., “Grid Analysis Using Extended Slope Deflection Method, the Bridge and Structural Engineer”, *IABSE, Vol. 28, No. 2, July 1998*, pp 1-10.
5. Pandit, G.S. and Gupta, S.P., “Structural Analysis – A Matrix Approach”, Tata McGraw Hill, 2007.